Pseudodifferential operators, 7.5 hp

Course period:
February 1 - May 4, 2019

Last day for application:
February 1, 2019

Course leader / Address for applications:
Magnus Goffeng / goffeng@chalmers.se

Course description (Advertisement for Ph.D. students):
A natural tool to study elliptic differential equations is the theory of pseudodifferential operators. The ideas of this theory goes back far, but important milestones is the usage of pseudodifferential operators in the K-theoretic proof of Atiyah-Singer's index theorem for elliptic differential operators and the work of Hörmander. The basic idea is to introduce a "calculus" in which one can carry out basic operations on elliptic differential operators modulo lower order error terms, e.g. computing an approximate inverse and constructing complex powers.

In this course we will study Hörmander's pseudodifferential calculus, first the local theory on $\mathbb{R}^n$ and then on a manifold. We will also consider the action of these operators on Sobolev spaces, in particular the functional analytic and spectral aspects of elliptic operators.

The course will start in February and run once a week (2 hours) until May (LP3-4). The period February to April is dedicated to lectures and May to student presentations. The schedule will be decided by participants at an introductory meeting.

Responsible department and other participation departments/organisations:
Mathematics Department

Teacher:
Magnus Goffeng

Examiner:
Magnus Goffeng

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1. Confirmation
The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX-XX.

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Elective course; third-cycle education

3. Entry requirements
Functional analysis, Higher differential calculus, Distribution theory

4. Course content
The course will cover the following topics.

- Oscillatory integrals and Hörmander symbols
- Pseudodifferential calculus
- Change of variables and pseudodifferential operators on manifolds
- L^2-boundedness: using calculus and Calderon-Vaillancourt's theorem
- Pseudodifferential operators and Sobolev spaces
- Fredholm properties and spectral properties of elliptic operators
- Solution methods for elliptic problems

5. Outcomes
At the end of the course, the students will have acquired knowledge about how to solve elliptic problems and some of the main results and techniques of pseudodifferential operators.

6. Literature
There are many good books on pseudodifferential operators, focussing on different aspects of the topic. The main reference for the course will be *Pseudodifferential Operators and Spectral Theory* by Mikhail Shubin, more specifically its first chapter. The books *The Analysis of Linear Partial Differential Operators III* by Lars Hörmander and *Harmonic analysis in phase space* by Gerald Folland.

7. Assessment
There will be a few homework sheets, and student presentations complemented by an oral exam at the end of the course

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail, (U), Pass (G)

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.